PI-CONTROL IN HYBRID FIRE TESTING

E. Mergny, G. Drion, J-M Franssen

INTRODUCTION

Hybrid testing has been successfully developed over the past decades in earthquake engineering and begins to be applied to fire engineering (Hybrid Fire Testing or HFT). The first attempt of HFT was done by Korzen *et al.* [1] and other researchers have followed suit. In most cases, an elastic response of the numerical substructure was considered. Yet, the numerical substructure can exhibit a nonlinear behaviour, notably if it is exposed to fire. This nonlinear response can be assessed by numerical simulation using a FEM model as presented in Mostafaei *et al.* [2] that uses a non-linear model for the numerical substructure.

Recently, Mergny *et al.* [3] proposes a framework based on linear control system theory for a displacement control procedure. It uses proportional integral controller to address identified stability issues and control the time properties in hybrid fire testing. However, the research did not address the issue of delay, error and sensitivity.

STATE SPACE REPRESENTATION OF HYBRID FIRE TESTING

Formally, [3] showed that HFT can be expressed as a discrete control problem and formulated the following state equation for the system:

$$\mathbf{u}_{\mathrm{PS}_{\mathrm{i+1}}} = -\mathbf{K}_{\mathrm{NS}_{\mathrm{i}}}^{-1}\mathbf{K}_{\mathrm{PS}_{\mathrm{i}}}\mathbf{u}_{\mathrm{PS}_{\mathrm{i}}} + \mathbf{u}_{\mathrm{NS}_{\mathrm{i}}}^{\mathrm{TH}} - \mathbf{K}_{\mathrm{NS}_{\mathrm{i}}}^{-1}\mathbf{F}_{\mathrm{PS}_{\mathrm{i}}}^{\mathrm{TH}} \tag{1}$$

$\mathbf{u}_{\mathrm{PS}}$ is the displacement vector of the tested specimen (imposed by actuators) and is the state variable. $\mathbf{K}_{\mathrm{PS}}$ and $\mathbf{K}_{\mathrm{NS}}$ are the stiffness matrices of the physical and numerical substructure (PS and NS). $\mathbf{F}_{\mathrm{PS}}^{\mathrm{TH}}$ and $\mathbf{u}_{\mathrm{NS}}^{\mathrm{TH}}$ are the thermal forces of PS and thermal displacements of NS. $-\mathbf{K}_{\mathrm{NS}_{\mathrm{i}}}^{-1}\mathbf{K}_{\mathrm{PS}_{\mathrm{i}}}$ is the dynamics matrix of the system. The HFT is stable if the module of the eigenvalues of this matrix is lower than 1. This condition is hardly never reached for multiple-DOF HFT. A PI-controller was consequently developed in [3]. The displacement $\mathbf{u}_{\mathrm{PS}}$ is corrected at every time step $\Delta \mathrm{t}_{\mathrm{i}}$:

$$\mathbf{u}_{\mathrm{PS}_{\mathrm{i+1}}} = \mathbf{u}_{\mathrm{PS}_{\mathrm{i}}} + \mathbf{L}_{\mathrm{P}}\mathbf{e}_{\mathrm{i}} + \mathbf{L}_{\mathrm{J}}\mathbf{J}_{\mathrm{i}} \tag{2}$$

$\mathbf{e}_{\mathrm{i}}=\left(\mathbf{u}_{\mathrm{NS}_{\mathrm{i}}} - \mathbf{u}_{\mathrm{PS}_{\mathrm{i}}}\right)$ is the instantaneous error and $\mathbf{J}_{\mathrm{i}}$ the sum of $\mathbf{e}_{\mathrm{i}}$ over time. $\mathbf{L}_{\mathrm{p}}$ and $\mathbf{L}_{\mathrm{J}}$ are gain matrices and are diagonal. As showed in [3], they are designed based on the location of the eigenvalues of the dynamics matrix of the state equation of the complete system:

$$\begin{bmatrix} \mathbf{u}_{\mathrm{PS}_{\mathrm{i+1}}} \\ \mathbf{J}_{\mathrm{i+1}} \end{bmatrix} = \begin{bmatrix} \mathbf{I} - \mathbf{L}_{\mathrm{p}}\mathbf{K}_{\mathrm{NS}_{\mathrm{i}}}^{-1}\mathbf{K}_{\mathrm{PS}_{\mathrm{i}}} - \mathbf{L}_{\mathrm{p}} & \mathbf{L}_{\mathrm{J}} \\ -\mathbf{K}_{\mathrm{NS}_{\mathrm{i}}}^{-1}\mathbf{K}_{\mathrm{PS}_{\mathrm{i}}} - \mathbf{I} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{\mathrm{PS}_{\mathrm{i}}} \\ \mathbf{J}_{\mathrm{i}} \end{bmatrix} + \begin{bmatrix} \mathbf{L}_{\mathrm{p}}\mathbf{u}_{\mathrm{NS}_{\mathrm{i}}}^{\mathrm{TH}} - \mathbf{L}_{\mathrm{p}}\mathbf{K}_{\mathrm{NS}_{\mathrm{i}}}^{-1}\mathbf{F}_{\mathrm{PS}_{\mathrm{i}}}^{\mathrm{TH}} \\ \mathbf{u}_{\mathrm{NS}_{\mathrm{i}}}^{\mathrm{TH}} - \mathbf{K}_{\mathrm{NS}_{\mathrm{i}}}^{-1}\mathbf{F}_{\mathrm{PS}_{\mathrm{i}}}^{\mathrm{TH}} \end{bmatrix} \tag{3}$$

The matrix $\mathbf{K}_{\mathrm{NS}}$ and an estimation of the matrix $\mathbf{K}_{\mathrm{PS}}$ are essential for determining the gain matrices.

DELAY

Delay $\Delta \mathrm{t}_{\mathrm{i}}$ is an issue in HFT because the temperature (and thus displacements) changes continuously and disturb the system. Its value depends on the speed of the actuators and the computing time and is variable. The dynamics matrix in equation (3) is independent of the delay and thus the stability is not affected. However, it should be low enough to capture the behaviour of the specimen.

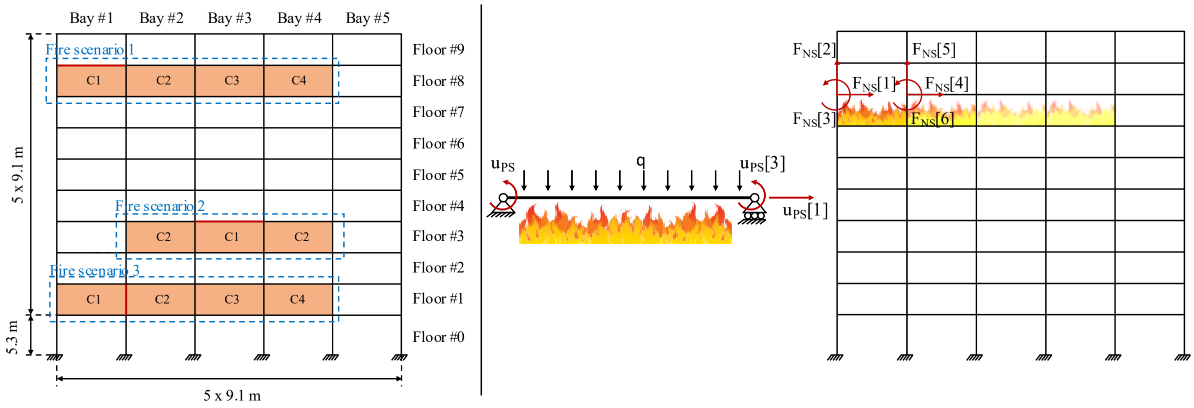
The value is thus limited by the Nyquist-Shannon theorem [4]: the sampling frequency should be at least twice the highest frequency contained in the signal, here $\mathbf{u}_{\mathrm{PS}}$. Initially, this input is not known. To apply this theorem, the minimum information that is necessary is the expected highest rate of $\mathbf{u}_{\mathrm{PS}}$ during the test. This condition must be verified before the HFT.

ERROR

Measurement errors will directly affect the equation (2) and (3): the correction is made based on altered values of $\mathbf{u}_{\mathrm{NS}_{\mathrm{i}}}$ and $\mathbf{u}_{\mathrm{PS}_{\mathrm{i}}}$. An error on the estimation of the stiffness of the PS $\mathbf{K}_{\mathrm{PS}}^{\mathrm{EST}}$ will influence the design of the gain matrices $\mathbf{L}_{\mathrm{p}}$ and $\mathbf{L}_{\mathrm{J}}$ and can be critical and lead to instability. The sensitivity of the method to this error must be evaluated.

APPLICATION

The sensitivity of the method to the estimation of the stiffness of PS is numerically tested on 3 fire scenarios performed with a multi-storey frame (Sadek *et al.* [5], Figure (a)). The hereunder results correspond to the first fire scenario showed on Figure 1 (b).



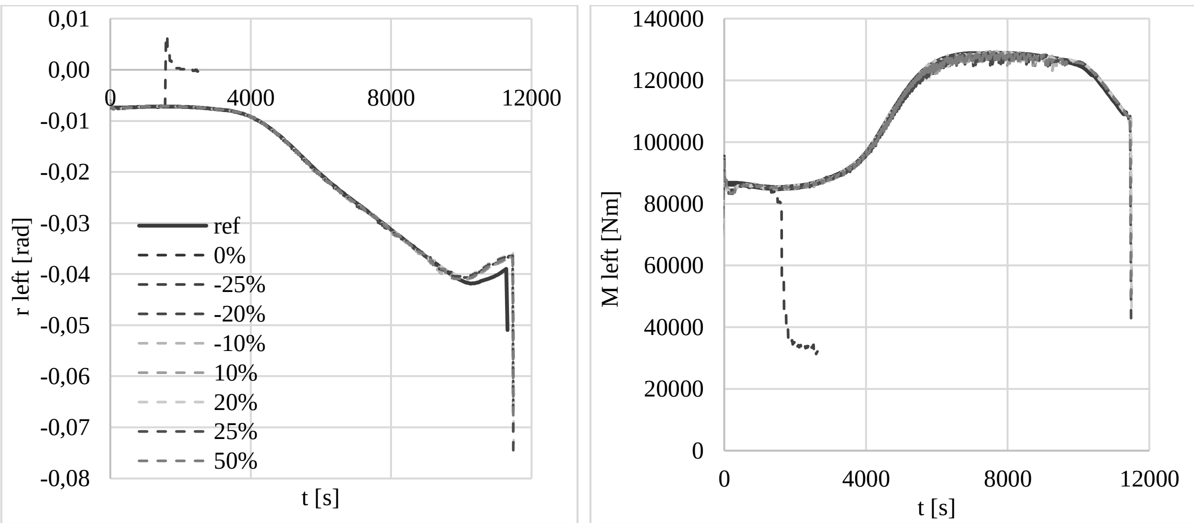
(a) (b)

Figure 1 – Multi-storey frame and fire scenarios (PS in red, C=Compartement) – PS and NS in fire scenario 1

A speed of 1mm/s was considered for the electrical actuators. The time for computing $\mathbf{u}_{\mathrm{NS}_i}$ is variable. It was estimated that the delay of 60 s would be sufficient to perform the application of the displacement and the numerical calculation. The hypothesis is made that the displacement rate is not higher than the temperature rate. The Nyquist-Shannon theorem is verified using this input.

A random error was introduced in the forces and displacements of the PS, considering uniform distribution that is the worst case. Overestimated and underestimated $\mathbf{K}_{\mathrm{PS}}^{\mathrm{EST}}$ are considered in %.

Results of the left rotation and left bending moment over time are shown on figure 2 (a) and (b) (Ref is the correct behaviour). They show that the stability and the accuracy of the solution for this scenario is not sensitive to overestimated and low underestimated stiffness. An underestimation higher than 25% is critical because the system becomes unstable. Results are similar for the two other scenarios.



(a) (b)

Figure 2 – Left rotation (a) – Left bending moment (b)

REFERENCES

[1] Korzen *et al.* 1999. Zeitschrift für Angewandte Mathematik und Mechanik.

[2] Mostafaei *et al.* 2013. Fire Safety Journal.

[3] Mergny *et al.* 2018. 10th International Conference on Structures in Fire SiF'18.

[4] Shannon. 1949. Proceedings of the Institute of Radio Engineers.

[5] Sadek *et al.* 2010. NIST Technical Note 1669.